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Journal of Geometry and Physics 49 (2004) 332–342
JOURNAL OF GEOMETRY AND PHYSICS
www.elsevier.com/locate/jgp

A new class of LRS Bianchi type-I cosmological models in Lyra geometry

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Received 5 November 2002; received in revised form 23 May 2003

Abstract

LRS Bianchi type-I models have been studied in the cosmological theory based on Lyra's geometry. A new class of exact solutions has been obtained by considering a time dependent displacement field for constant deceleration parameter models of the universe. The physical behaviour of the models is examined in vacuum and in the presence of perfect fluids.

MSC: 83C20

PACS: 98.80

Subj. Class.: General relativity

Keywords: Cosmology; LRS Bianchi type-I models; Lyra geometry

1. Introduction

In 1917 Einstein introduced the cosmological constant into his field equations in order to obtain a static cosmological model since, as is well known, without the cosmological term his field equations admit only non-static solutions. After the discovery of the redshift of galaxies and its explanation as being due to the expansion of the universe, Einstein regretted his introduction of the cosmological constant. Recently, there has been much interest in the cosmological term in context of quantum field theories, quantum gravity, supergravity theories, Kaluza–Klein theories and the inflationary-universe scenario. Shortly after Einstein's

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0393-0440/$ – see front matter
doi:10.1016/S0393-0440(03)00105-0

general theory of relativity Weyl, in 1918, suggested the first so-called unified field theory based on a generalization of Riemannian geometry. In retrospect, it would seem more appropriate to call Weyl's theory a geometrized theory of gravitation and electromagnetism (just as the general theory was a geometrized theory of gravitation only), rather than a unified field theory. It is not quite clear to what extent the two fields have been unified, even though they acquire (different) geometrical significances in the same geometry. The theory was never taken seriously because it was based on the concept of non-integrability of length transfer, and, as pointed out by Einstein, this implies that spectral frequencies of atoms depend on their past histories and therefore have no absolute significance. Nevertheless, Weyl's geometry provides an interesting example of non-Riemannian connections, and recently Folland [1] has given a global formulation of Weyl manifolds thereby clarifying considerably many of Weyl's basic ideas.

In 1951 Lyra [2] proposed a modification of Riemannian geometry by introducing a gauge function into the structureless manifold, as a result of which the cosmological constant arises naturally from the geometry. This bears a remarkable resemblance to Weyl's geometry. But in Lyra's geometry, unlike Weyl's, the connection is metric preserving as in Remannian; in other words, length transfers are integrable. Lyra also introduced the notion of a gauge and in the "normal" gauge the curvature scalar in identical to that of Weyl. In consecutive investigations, Sen [3], Sen and Dunn [4] proposed a new scalar–tensor theory of gravitation and constructed an analogue of the Einstein field equations based on Lyra's geometry. It is thus possible [3] to construct a geometrized theory of gravitation and electromagnetism much along the lines of Weyl's "unified" field theory without, however, the inconvenience of non-integrability length transfer.

Halford [5] has pointed out that the constant vector displacement field ϕ_i in Lyra's geometry plays the role of cosmological constant Λ in the normal general relativistic treatment. It is shown by Halford [6] that the scalar–tensor treatment based on Lyra's geometry predicts the same effects, within observational limits, as the Einstein's theory. Several authors Sen and Vanstone [7], Bhamra [8], Karade and Borikar [9], Kalyanshetti and Waghmode [10], Reddy and Innaiah [11], Beesham [12], Reddy and Venkateswarlu [13], Soleng [14], have studied cosmological models based on Lyra's manifold with a constant displacement field vector. However, this restriction of the displacement field to be constant is merely one of convenience and there is no a priori reason for it. Beesham [15] considered FRW models with time dependent displacement field. He has shown that by assuming the energy density of the universe to be equal to its critical value, the models have the $k = -1$ geometry. Singh and Singh [16–19], Singh and Desikan [20] have studied Bianchi type-I, -III, Kantowaski–Sachs and a new class of cosmological models with time dependent displacement field and have made a comparative study of Robertson–Walker models with constant deceleration parameter in Einstein's theory with cosmological term and in the cosmological theory based on Lyra's geometry. Soleng [14] has pointed out that the cosmologies based on Lyra's manifold with constant gauge vector ϕ will either include a creation field and be equal to Hoyle's creation field cosmology [21–23] or contain a special vacuum field which together with the gauge vector term may be considered as a cosmological term. In the latter case the solutions are equal to the general relativistic cosmologies with a cosmological term.

The Einstein's field equations are a coupled system of highly non-linear differential equations and we seek physical solutions to the field equations for their applications in cosmology and astrophysics. In order to solve the field equations we normally assume a form for the matter content or that space–time admits killing vector symmetries [24]. Solutions to the field equations may also be generated by applying a law of variation for Hubble's parameter which was proposed by Berman [25]. It is interesting to obverse that the law yields a constant value for deceleration parameter. The variation of Hubble's law as assumed is not inconsistent with observation and has the advantage of providing simple functional forms of the scale factor. In simplest case the Hubble law yields a constant value for the deceleration parameter. It is worth observing that most of the well-known models of Einstein's theory and Brans–Deke theory with curvature parameter $k = 0$, including inflationary models, are models with constant deceleration parameter. In earlier literature cosmological models with a constant deceleration parameter have been studied by Berman [25], Berman and Gomide [26], Johri and Desikan [27], Singh and Desikan [20], Maharaj and Naidoo [28], Pradhan et al. [29] and others. This has provided us the motivation to study models with constant deceleration parameter in Lyra geometry.

At the present state of evolution, the universe is spherically symmetric and the matter distribution in it is on the whole isotropic and homogeneous. But in its early stages of evolution, it could not have had such a smoothed out picture. Close to the big bang singularity, neither the assumption of the spherically symmetric nor of isotropy can be strictly valid. So we consider plane symmetry which is less restrictive than spherical symmetry and provides an avenue to study inhomogeneities. The present investigation is concern with local rotational symmetry (LRS) Bianchi type-I cosmological model with both cases, viz., time dependent and constant displacement vectors based on Lyra's geometry in normal gauge. All observations at every general point are rotationally symmetric about this direction. It may be pointed out that similar results can be obtained in several other theories as well as Einstein's theory minimally coupled to a massless scalar field and in Hoyle's creation field theory [21], if the creation field is assumed to be time dependent. Such investigations have not been undertaken in Hoyle's theory so far.

2. Field equations

We consider LRS Bianchi type-I space–time

$$\mathrm{d}s^2 = \mathrm{d}t^2 - A^2\,\mathrm{d}x^2 - B^2(\mathrm{d}y^2 + \mathrm{d}z^2), \tag{1}$$

where $A = A(x,t)$, $B = B(x,t)$. We take a perfect fluid form for the energy momentum tensor

$$T_{ij} = (p+\rho)u_i u_j - p g_{ij} \tag{2}$$

together with comoving coordinates $u^i u_i = 1$, where $u_i = (0, 0, 0, 1)$.

The field equations in normal gauge for Lyra's manifold, as obtained by Sen [3] are

$$R_{ij} - \tfrac{1}{2} g_{ij} R + \tfrac{3}{2}\phi_i\phi_j - \tfrac{3}{4} g_{ij}\phi_k\phi^k = -8\pi G T_{ij}, \tag{3}$$

where ϕ is a time-like displacement field vector defined by $\phi_i = (0, 0, 0, \beta(t))$ and other symbols have their usual meaning as in Riemannian geometry. Here we want to mention the fact that the ansatz choosing the coordinate system with matter require the vector field happens to be in the required form exactly in the matter comoving coordinates. The essential difference between the cosmological theories based on Lyra geometry and the Riemannian geometry lies in the fact that constant vector displacement field β arises naturally from the concept of gauge in Lyra geometry where as cosmological constant Λ was introduced in ad hoc fashion in the usual treatment. Now the field equations can be set up and one obtains

$$\frac{2\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} - \frac{B'^2}{A^2B^2} + \frac{3}{4}\beta^2 = -\chi p, \tag{4}$$

$$\dot{B}' - \frac{B'\dot{A}}{A} = 0, \tag{5}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{B''}{A^2B} + \frac{A'B'}{A^3B} + \frac{3}{4}\beta^2 = -\chi p, \tag{6}$$

$$\frac{2B''}{A^2B} - \frac{2A'B'}{A^3B} + \frac{B'^2}{A^2B^2} - \frac{2\dot{A}\dot{B}}{AB} - \frac{\dot{B}^2}{B^2} + \frac{3}{4}\beta^2 = \chi\rho. \tag{7}$$

The energy conservation equation is

$$\chi\dot{\rho} + \frac{3}{2}\beta\dot{\beta} + \left[\chi(p+\rho) + \frac{3}{2}\beta^2\right]\left(\frac{\dot{A}}{A} + \frac{2\dot{B}}{B}\right) = 0, \tag{8}$$

where $\chi = 8\pi G$. Here and in what follows a prime and a dot indicate partial differentiation with respect to x and t, respectively. Eqs. (4)–(7) are four equations in five unknowns A, B, β, p and ρ. For complete determinacy of the system one extra condition is needed. One way is to use an equation of state. The other alternative is a mathematical assumption on the space–time and then to discuss the physical nature of the universe. In this paper we confine ourselves to assume an equation of state

$$p = \gamma\rho, \quad 0 \le \gamma \le 1. \tag{9}$$

3. Solutions of the field equations

Eq. (5), after integration, yields

$$A = \frac{B'}{l}, \tag{10}$$

where l is an arbitrary function of x. Eqs. (4) and (6), with the use of Eq. (10), reduces to

$$\frac{B}{B'}\left(\frac{\ddot{B}}{B}\right)' + \frac{\dot{B}}{B'}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{B'}{B}\right) + \frac{l^2}{B^2}\left(1 - \frac{Bl'}{B'l}\right) = 0. \tag{11}$$

To get solution, let us assume

$$\frac{B'}{B} = \text{functions of } x. \tag{12}$$

By this choice, Eq. (11) gives after integration

$$B = lS(t), \tag{13}$$

where $S(t)$ is an arbitrary function of t. With the help of Eq. (13), Eq. (10) becomes

$$A = \frac{l'}{l}S. \tag{14}$$

Now the metric (1) takes the form

$$\mathrm{d}s^2 = \mathrm{d}t^2 - S^2(t)[\mathrm{d}X^2 + \mathrm{e}^{2X}(\mathrm{d}y^2 + \mathrm{d}z^2)], \tag{15}$$

where $X = \ln l$. Eqs. (4) and (7) give

$$\chi p = \frac{1}{S^2} - 2\frac{\ddot{S}}{S} - \frac{\dot{S}^2}{S^2} - \frac{3}{4}\beta^2, \tag{16}$$

$$\chi \rho = \frac{3\dot{S}^2}{S^2} - \frac{3}{S^2} - \frac{3}{4}\beta^2. \tag{17}$$

Using Eq. (9) and eliminating $\rho(t)$ from Eqs. (16) and (17) we have

$$\frac{2\ddot{S}}{S} + (1+3\gamma)\frac{\dot{S}^2}{S^2} - (1+3\gamma)\frac{1}{S^2} + \frac{3}{4}(1-\gamma)\beta^2 = 0. \tag{18}$$

Now the expressions for the energy density and the pressure are given by

$$\chi p = \chi\gamma\rho = \frac{4\gamma}{1-\gamma}\left[\frac{\dot{S}^2}{S^2} - \frac{1}{S^2} + \frac{\ddot{S}}{2S}\right]. \tag{19}$$

The function $S(t)$ remains undetermined. To obtain its explicit dependence on t, one may have to introduce additional assumptions. In the following we assume the deceleration parameter to be constant to achieve this objective, i.e.

$$q = -\frac{S\ddot{S}}{\dot{S}^2} = -\left(\frac{\dot{H} + H^2}{H^2}\right) = b(\text{constant}), \tag{20}$$

where $H = \dot{S}/S$ is the Hubble parameter. The sign of deceleration parameter q indicates whether the cosmological model inflates. The positive sign corresponds to "standard" decelerating models whereas the negative sign indicates inflation. To study inflationary models we assume $q =$ constant. This Eq. (20) is integrated to obtain

$$S(t) = \begin{cases} [a(t-t_0)]^{1/(1+b)} & \text{when } b \neq -1, \\ m_1\,\mathrm{e}^{m_2 t} & \text{when } b = -1, \end{cases} \tag{21}$$

where a, m_1 and m_2 are constants of integration and the constant t_0 means the freedom of choosing the time origin. Using Eq. (20) into Eqs. (18) and (19) lead to

$$\beta^2 = \frac{4}{3(1-\gamma)}\left[(1+3\gamma)\frac{1}{S^2} - (1+3\gamma-2b)H^2\right], \tag{22}$$

Table 1
Values of β^2 and ρ for dust and radiation power-low models

γ	β^2	ρ
0	$\frac{4}{3}\left[\frac{1}{S^2} - (1-2b)H^2\right]$	$\frac{2}{\chi}\left[(2-b)H^2 - \frac{2}{S^2}\right]$
$\frac{1}{3}$	$4\left[\frac{1}{S^2} - (1-b)H^2\right]$	$\frac{3}{\chi}\left[(2-b)H^2 - \frac{2}{S^2}\right]$

$$\chi\rho = \frac{1}{1-\gamma}\left[2(2-b)H^2 - \frac{4}{S^2}\right]. \tag{23}$$

The expressions for β^2 and ρ corresponding to $\gamma = 0, 1/3$ are summarized in Table 1. Now we consider some physically interesting particular cases.

3.1. Flat models

The condition for spatially flat model is obtained as

$$\frac{1}{S^2} = (1-b)H^2. \tag{24}$$

Using Eq. (24), Eqs. (22) and (23) reduce to

$$\beta^2 = \frac{4(3\gamma-1)}{3(\gamma-1)}bH^2 \tag{25}$$

and

$$\chi\rho = -\frac{2bH^2}{\gamma-1}. \tag{26}$$

From Eq. (26) we see that $\rho \geq 0$ if $1 > b > 0$ since $(\gamma - 1) < 0$. From Eq. (25) we see that since $(\gamma - 1) < 0$, $\beta^2 > 0$ if $\gamma < 1/3$ and $\beta^2 < 0$ if $\gamma > 1/3$. The expressions for β^2 and ρ corresponding to $\gamma = 0, 1/3$ are given in Table 2.

Here we observe that when $\gamma = 1/3$, $\beta^2 = 0$ and the equations reduce to those of LRS Bianchi type-I flat universe. For $b = -1$, the energy density always comes negative. So we only consider the case $b \neq -1$.

Case (i): $b \neq -1$. For singular models, Eq. (17) leads to

$$S = mt^{1/(1+b)}. \tag{27}$$

Table 2
Values of β^2 and ρ for dust and radiation power-law models

γ	β^2	ρ
0	$\frac{4}{3}bH^2$	$2bH^2$
$\frac{1}{3}$	0	$3bH^2$

Using Eq. (27) into Eqs. (25) and (26) yield

$$\beta^2 = \frac{4(3\gamma-1)}{3(\gamma-1)}\frac{b}{(1+b)^2t^2} \tag{28}$$

and

$$\chi\rho = -\frac{2b}{(\gamma-1)(1+b)^2t^2}. \tag{29}$$

The above expressions for β^2 and energy density $\rho(t)$ are similar to those obtained by Beesham [30] for a variable cosmological term $\wedge(t)$ and energy density $\rho(t)$. Here β^2 plays the role of a variable cosmological term $\wedge$. Eqs. (9) and (29) give

$$\rho + 3p = \frac{-2(1+3\gamma)}{\chi(\gamma-1)}\frac{b}{(1+b)^2t^2}. \tag{30}$$

It can be seen from the above expression that the condition $\rho + 3p \geq 0$ would hold only for $1 + 3\gamma \geq 0$, i.e. $\gamma \geq -1/3$. So for values of $\gamma < -1/3$, we cannot have viable models.

We observe from (28) and (29) that β^2 and ρ fall off as $1/t^2$ irrespective of the equation of state. The expressions for β^2 and ρ corresponding to $\gamma = 0, 1/3$ are given in Table 3. It can be easily seen from Eqs. (28) and (29) or otherwise also that the expressions for β^2 and ρ will not be valid for the empty universe (i.e. $p = \rho = 0$) and the stiff matter (i.e. $p = \rho$) models. So we shall not discuss these models.

3.2. Non-flat models

Case (i): $b \neq -1$. Using Eq. (27) into Eqs. (22) and (23) lead to

$$\beta^2 = \frac{4}{3(\gamma-1)}\left[\frac{1+3\gamma-2b}{(1+b)^2t^2} + \frac{1+3\gamma}{m^2t^{2/(1+b)}}\right] \tag{31}$$

and

$$\chi\rho = \frac{2}{\gamma-1}\left[\frac{b-2}{(1+b)^2t^2} + \frac{2}{m^2t^{2/(1+b)}}\right]. \tag{32}$$

From Eq. (32), we see that $\rho \geq 0$ when $-1 < b < 2$ as $(\gamma - 1) < 0$. From Eq. (30), we see that for $b < (1 + 3\gamma)/2$, $\beta^2 < 0$ for all times $t > 0$ as $(\gamma - 1) < 0$. It is also observed that for $(1 + 3\gamma)/2 < b \leq 2$, $\beta^2 > 0$ for

$$0 < t^{2b/(1+b)} < \frac{(2b-1-3\gamma)m^2}{(1+3\gamma)(1+b)^2} \tag{33}$$

Table 3
Values of β^2 and ρ for dust and radiation power-law models

γ	β^2	ρ
0	$\frac{4b}{3(1+b)^2t^2}$	$\frac{2b}{(1+b)^2t^2}$
$\frac{1}{3}$	0	$\frac{3b}{(1+b)^2t^2}$

and $\beta^2 < 0$ for

$$t^{2b/(1+b)} > \frac{(2b-1-3\gamma)m^2}{(1+3\gamma)(1+b)^2}. \tag{34}$$

At

$$t^{2b/(1+b)} = \frac{(2b-1-3\gamma)m^2}{(1+3\gamma)(1+b)^2}, \quad \beta^2 = 0. \tag{35}$$

When $b = (1+3\gamma)/2$, Eqs. (31) and (32) reduce to

$$\beta^2 = \frac{4(1+3\gamma)}{3(\gamma-1)m^2 t^{4/3(1+\gamma)}} \tag{36}$$

and

$$\chi\rho = \frac{1}{1-\gamma}\left[\frac{3(1-\gamma)}{(1+b)^2 t^2} - \frac{4}{m^2 t^{4/3(1+\gamma)}}\right]. \tag{37}$$

From Eq. (36), it is obvious that $\beta^2 < 0$ for all times as $(\gamma - 1) < 0$. The expressions for β^2 and ρ cannot be determined for the empty universe (i.e. $p = \rho = 0$) and stiff matter $(p = \rho)$ models. The above expressions for ρ and β^2 are similar to those of ρ and β^2 given by Eqs. (35)–(38) of Singh and Desikan [20].

Physical behaviour of the model. In the case of a non-flat model when $b \neq -1$, the Ricci scalar becomes

$$R = \frac{1}{m^2 t^{2/(1+b)}} - \frac{1-b}{1+b}t. \tag{38}$$

It is observed from Eq. (38) that when $t \to 0$ (i) $R \to \infty$ if $b = 0$, (ii) $R \to \infty$ if $b \geq 1$ and (iii) $R \to \infty$ if $b \leq -2$. Eq. (38) also suggests that when $t \to \infty$ (i) $R \to 0$ if $b \geq 0$ and (ii) $R \to \infty$ if $b \leq -2$.

The scalars of expansion and shear are given by

$$\theta = \frac{3}{(1+b)t}, \qquad \sigma = 0. \tag{39}$$

The model has singularity at $t = 0$. At $t \to \infty$, the expansion ceases. The gauge function β was large in the beginning but decreases fast with the evolution of the model. Similar results can be obtained for Hoyle's creation field theory if the creation field is time dependent. Here $\sigma/\theta = 0$, which confirms the isotropic nature of the space–time which we have already obtained in Eq. (15).

Case (ii): $b = -1$. In this case, Eq. (20) becomes

$$\dot{H} = 0 \quad \text{and} \quad H = H_0 = \text{constant}. \tag{40}$$

Using Eq. (40) into Eqs. (22) and (23) we have

$$\beta^2 = \frac{4}{3(\gamma-1)}\left[3(1+\gamma)H_0^2 - \frac{1+3\gamma}{m_1^2}\mathrm{e}^{-2m_2 t}\right], \tag{41}$$

Table 4
Values of β^2 and ρ for dust and radiation exponential

γ	β^2	ρ
0	$-\frac{4}{3}\left[3H_0^2 - \frac{e^{-2m_2 t}}{m_1^2}\right]$	$2\left[3H_0^2 - \frac{2\,e^{-2m_2 t}}{m_1^2}\right]$
$\frac{1}{3}$	$-4\left[2H_0^2 - \frac{e^{-2m_2 t}}{m_1^2}\right]$	$3\left[3H_0^2 - \frac{2\,e^{-2m_2 t}}{m_1^2}\right]$

$$\chi\rho = \frac{2}{1-\gamma}\left[3H_0^2 - \frac{2}{m_1^2}e^{-2m_2 t}\right]. \tag{42}$$

From Eq. (42), we observe that $\rho > 0$ only for $H_0^2 > 2/3m_1^2$ as $(1-\gamma) > 0$. $\beta^2 < 0$ for all times as can be seen from Eq. (41) as $(\gamma - 1) < 0$. For large times, i.e. $t \to \infty$ we see that β^2 and ρ would reach steady state, i.e.

$$\beta^2 \to \frac{4}{\gamma - 1}(1+\gamma)H_0^2 \quad \text{and} \quad \chi\rho \to \frac{6}{1-\gamma}H_0^2. \tag{43}$$

The strong energy condition, as mentioned by Ellis [31], $\rho + 3p > 0$ is also satisfied for $H_0^2 > 2/3m_1^2$. The expressions for β^2 and ρ corresponding to $\gamma = 0, 1/3$ are given in Table 4.

3.3. *Empty universe*

In the case of empty universe $(p = \rho = 0)$ Eq. (42) reduces to

$$3H_0^2 - \frac{2}{m_1^2}e^{-2m_2 t} = 0. \tag{44}$$

Using Eq. (44) into Eq. (41) leads to

$$\beta^2 = -2H_0^2. \tag{45}$$

For H_0 to be real, β must be imaginary.

Physical behaviour of the model. The Ricci scalar R is

$$R = 2H_0^2 - \frac{1}{m_1^2}e^{-2m_2 t}. \tag{46}$$

It is easy to see that (i) when $t \to 0$, $R \to 2H_0^2 - 1/m_1^2$ and (ii) when $t \to \infty$, $R \to 2H_0^2$ when $m_2 > 0$ and $R \to \infty$ when $m_2 < 0$. The expansion and shear scalars are

$$\theta = 3H_0, \qquad \sigma = 0. \tag{47}$$

The model represents an uniform expansion as can be seen from Eq. (47). The flow of the fluid is geodetic as the acceleration vector $f_i = (0, 0, 0, 0)$.

4. Conclusions

In this paper we have obtained exact solutions of Sen equations in Lyra geometry for constant deceleration parameter. The nature of the displacement field $\beta(t)$ and the energy density $\rho(t)$ have been examined for both the (i) power-law and (ii) exponential expansion of both the flat universe and the non-flat universe. The solutions obtained in Sections 3.1 and 3.3 are quite new solutions. Here the displacement field β plays the role of a variable cosmological term Λ.

Recently, there is an upsurge of interest in scalar fields in general relativity and alternative theories of gravitation in the context of inflationary cosmology. Therefore, the study of cosmological models in Lyra geometry may be relevant for inflationary models. Further the space dependence of the displacement field β is important for inhomogeneous models for the early stage of the evolution of the universe. Besides, the implication of Lyra's geometry for astrophysical interesting bodies is still an open question. The problem of equations of motion and gravitational radiation need investigation.

Acknowledgements

The authors would like to thank the Inter-University Centre for Astronomy and Astrophysics, Pune, India, where this work was carried out, for providing them its facilities. Authors express their gratitude to T. Singh and G.P. Singh for many helpful discussions. The authors are also thankful to the referee for his constructive comments for the improvement of the paper.

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